

A classroom design challenge inspired by Kathak

Rhythm, Shape & सम

Make a rhythm. Map a shape.
Explain the mathematics.

Classes 6-8

3 × 40 minutes

Teams of 3-4

Paper first

Design a short rhythm score with a steady 16-beat cycle, two subdivision choices, and a three-part ending that returns to सम (sam, pronounced “sum”), the first beat. Add a diagram that explains a reflection or rotation. A silent paper demonstration is a complete presentation.

Prepare in five minutes

- Read the counting convention on page 2.
- Print learner pages 7-9, or copy their prompts onto plain paper.
- Gather pencils, paper, 16 paper tokens per team and a paper arrow.
- Draw a 16-box beat strip on the board; divide it into four groups of four.

What learners will explain

- How 24 taps can fit into 16 beats.
- How to work backward from a target beat.
- Why 90° and 450° can finish in the same direction.
- How to check a design using evidence.

Kathak is a living Indian classical dance tradition, including storytelling, rhythm, music and expressive movement. This is a mathematics exploration inspired by selected structures. The tapping is not formal tatkar; the paper ending is not a traditional composition; the arrow task does not teach chakkar technique. A trained practitioner is needed for dance instruction.

Based on the article “*Kathak and Mathematics: Exploring the Mathematical Foundations of Rhythm, Geometry, and Movement*” by **Sohini Bandopadhyay**. The practical classroom sequence, diagrams, interactive tools and teacher resources were developed for Zalki.

No dance experience, costumes, musical instruments, internet connection or individual public performance is required. Online tools are optional: zalki.com/kathak.html.

Keep the meaning of each count visible

Before the first beat

Words for this project

- **Matra**: one equal beat unit in our classroom model.
- **Teentaal**: 16 beats grouped 4 + 4 + 4 + 4.
- **सम**: beat 1; the beginning and an important point of return.
- **Subdivision**: equal taps inside one underlying beat.
- **Khali**: the section marked by a wave at beat 9; it is not an instruction to remove its beats or sounds.

Mark the cycle

Count 1-16. Clap at **1, 5, 13**; wave at **9**. Learners can point or move a token instead. Keep the marker row separate from the tap pattern.

The first four whole beat durations are $\frac{1}{4}$ of the cycle. Their endpoint is the start of beat **5**. Half the cycle has elapsed when beat **9** begins.

Each box is a beat-position slot. **o** means one tap at the beat's onset. **-** means a silent slot. Three equal phrases with two equal internal gaps form our **tihai-inspired ending**. Its last tap lands at the onset of the next सम.

o o o - o o o - o o o
7 8 9 10 11 12 13 14 15 16 next1

11 slots; 10 intervals from first tap to last. At 60 BPM, the last tap occurs 10 seconds after the first. Do not confuse this with eleven elapsed seconds. Other musical notation conventions need their own interpretation.

Participation choices for everyone

Offer clapping, quiet seated finger or desk tapping, moving a paper token, pointing, counting, drawing, checking or explaining. No learner has to spin, stamp, stand, touch a partner, make a loud sound or perform alone. Do not require learners to copy dance postures.

Keep the numbered strip visible. Use a larger, high-contrast strip, written or spoken explanations, partner descriptions or a silent visual count as appropriate. Every mathematical goal can be shown on paper. Assess the mathematics, not hearing, balance, movement range, rhythm precision or confidence.

Offline route

Use paper strips and tokens for every tool. A teacher's spoken count replaces playback. Grid sketches replace geometry tools. Copy the learner prompts if printing is unavailable. Choose 1 and 2 taps per beat; four subdivisions can stay a calculation.

Optional timing

Use a comfortable pulse, about 60 BPM. To time one cycle, measure from one beat-1 onset to the **following beat-1 onset**: 16 intervals. Beat 1 to beat 16 contains only 15 intervals.

Opening script: "We are using mathematics to notice some structures in Kathak. We will make our own classroom scores; we are not learning a complete dance form in three lessons."

Teacher sources · Cultural context & rhythmic conventions

Kathak Kendra · courses · CCRT · Kathak cultural package · Ravi Shankar Opera Project · resources · David Courtney · Tintal · NCERT · Learning Outcomes, Dance, printed pp. 107-108.

40 minutes · Cycle tool optional

Keep the beat. Change what fits inside it.

Learn: equal grouping, fractions and subdivision. **Collect:** a labelled 16-box strip, tap predictions and an individual exit explanation.
Use: learner page 7.

Min.	What to do · What to ask · Answers
0-4	Introduce Kathak and the design challenge. Offer the participation choices. Ask: "Sixteen counters in four equal groups: how many per group?" 4, because $16 \div 4 = 4$. Use this as a starting check.
4-8	Count 1-16, pointing at the strip. Clap at 1, 5, 13; wave at 9. Repeat once. "Does waving at 9 remove a beat?" No. Beat 9 and its whole four-beat group still count.
8-14	Pairs partition and label their strip: 4/16, 8/16, 12/16, 16/16. "After four whole beats have elapsed, which beat starts?" $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 whole. Beat 5 starts after $\frac{1}{4}$; beat 9 after $\frac{1}{2}$.
14-22	Keep a slow steady underlying beat. Try one tap per beat, then two evenly spaced taps: "1-and, 2-and..." Predict totals for 1, 2, 3 and 4 taps per beat; only perform 1 and 2. "What stays fixed?" 16, 32, 48, 64 taps. The underlying 16 beats and cycle duration stay fixed at the same tempo. At 60 BPM, two subdivisions are $\frac{1}{2}$ second apart; four are $\frac{1}{4}$ second apart.
22-30	Teams design an opening: successive four-beat groups have 1, 2, 1, 2 taps per beat. Draw dots inside each beat box. Rotate counter, score maker and checker roles. "How many taps?" $4 \times 1 + 4 \times 2 + 4 \times 1 + 4 \times 2 = 24$ taps in 16 underlying beats.
30-35	Pairs exchange strips. Check: 16 boxes? Four equal groups? Dot total correct? Two taps inside the same box? Record a confirmation or correction. Timing can be checked by pointing.
35-40	Exit: "A 16-beat cycle has two taps per beat. How many taps? Does it now have twice as many beats? Which beat starts at the halfway point?" 32 taps; still 16 beats; beat 9. Accept a diagram plus explanation.

Make it accessible

Use four groups of tokens. Keep 1 and 2 subdivisions only. One learner can place dots while a partner counts. A complete drawn score is sufficient.

35-40 minutes

For 35: design for five minutes and peer-check for three, retaining the five-minute exit. For 38: peer-check for three minutes.

Extend the reasoning

Use 1, 2, 3, 4 taps in successive groups: $4(1+2+3+4) = 40$. At 80 BPM, one beat lasts **0.75 s**, one cycle **12 s**. At 120 BPM: **0.5 s** and **8 s**.

Watch for: "Beat 4 is a quarter" is ambiguous. Four complete beat durations end at the onset of beat 5.

40 minutes · Tihai tool optional

Design a three-part return to सम.

Learn: repeated phrases, counted gaps and working backward. **Collect:** a labelled trace to next सम and a reason for its starting beat. **Use:** learner page 8. Add a **next1** box after beat 16.

Min.	What to do · What to ask · Answers
0-4	Retrieve beat versus subdivision. Count "...15, 16, 1, 2." "What follows 16? Is सम a pause?" Beat 1 of the next cycle. सम is the first beat, not a pause.
4-9	Build o o o - o o o - o o o . Each dot is a tap on a beat; each dash is a whole silent beat-position slot. Ask for copies, gaps and total slots. 3 equal phrases; 2 internal gaps; $3 \times 3 + 2 \times 1 = 11$ slots. No extra gap follows the last phrase.
9-17	Place the last tap on next1. Count backward one box at a time, then read the labelled strip forward. Learners use taps, dots or tokens. Tap 7,8,9; rest10; tap 11,12,13; rest14; tap 15,16,next1. Start 7. Eleven positions contain ten first-to-last intervals.
17-23	Test an error: "I start on 6 because $16-11+1 = 6$." Write all eleven beat labels and correct the claim. 6 through 16 gives eleven positions, but lands on 16: one beat early. The target is next1, so start 7.
23-31	Teams add the ending to their score. Count six listening/pointing beats, then enter on 7. "Why is 10 silent? Does counting stop?" Each learner explains one choice. 10 is the gap after phrase one; counting continues through silence. The labelled trace proves the final arrival.
31-35	Exchange scores. Check equal phrases, two equal gaps, labelled silent slots and final next1. Record "verified" or a specific correction.
35-40	Exit: "L = 2 taps per phrase; G = 1 silent slot between phrases. How many slots? Start where to end on next1?" Show every label. $3 \times 2 + 2 \times 1 = 8$ slots. Start10: 10,11 / rest12 / 13,14 / rest15 / 16,next1.

For this whole-beat slot model, $N = 3L + 2G$ slots. Count backward **N-1 intervals** from next1. For $3 \leq N \leq 17$, start = 18-N; verify with a trace.

L3,G0: 9 slots, start9. **L3,G1:** 11 slots, start7.

L3,G2: 13 slots, start5. **L4,G1:** 14 slots, start4.

Support: provide labels 7,8,9,10,11,12,13,14,15,16,1; learners add the dot/dash strip.

35 minutes: test the error for four minutes and design for five. **38:** test the error for four minutes. Retain the five-minute exit.

Watch for: 11 slots do not mean 11 elapsed intervals to the final onset. At 60 BPM the final tap is **10 seconds** after the first tap. The final slot continues after that onset if time is counted further.

40 minutes · Geometry tool optional

Map shapes. Show the reasoning.

Learn: angles, final direction, reflection and mathematical explanation. **Collect:** the completed score, a geometry diagram, a peer check and individual reflection. **Use:** learner page 9.

Min.	What to do · What to ask · Answers
0-5	Paper arrow points N on an overhead compass diagram. Rotate clockwise in quarter-turns. "If it ends N, must it have made zero turns?" No body turns are needed. 90°→E; 180°→S; 270°→W; 360°→N. Ending N can follow 0°, 360°, 720°...
5-12	Teams draw a rotation: start direction, turn direction, total angle and final direction. Make another with the same final direction and a different total turn. 90° and 450° clockwise from N both end E. $450^\circ = 360^\circ + 90^\circ$. $3 \times 360^\circ = 1080^\circ$ ends N. Say "total turn," not "net displacement."
12-19	Draw vertical mirror line $x=0$. Reflect (-2,1), (-1,2), (-2,-1), (-1,-2) using tokens or pencil dots. Mark equal perpendicular distances to the line. (2,1), (1,2), (2,-1), (1,-2). Height stays fixed. (-2,1)→(2,-1) is wrong for this mirror because it changes the height.
19-27	Finish the opening, ending and one geometry diagram. Write two "because" statements. Display the diagram with the rhythm, or point through the whole score silently. An optional run-through takes about 32 seconds at 60 BPM. Default opening: 24 taps. Ending: start7, finish next1. Geometry: name the transformation and a measurable fact.
27-35	Pair teams: A explains for 3 minutes, B checks; swap for 3 minutes, then revise for 2. Ask: "Where are the 16 beats? Prove the final landing. Show an angle or equal mirror distances. What changed after checking?" Teacher samples explanations.
35-40	Individual reflection: "Explain one team decision using a calculation or diagram. Name one part of Kathak this project does not teach." Examples: $24=4+8+4+8$; start7 trace; $450^\circ=360^\circ+90^\circ$. Limits include dance technique, formal tatkar, chakkars and expressive storytelling.

Support & time options

Use 90°, 180°, 270°, 360° only; reflect one point at a time. Give N/E/S/W labels and allow a partner to read an explanation.

For 35 minutes: finish the score in five minutes; gallery teams present for two minutes each, then revise for two. For 38: finish in six minutes. Keep the five-minute reflection.

Extend: order 4

Place identical round tokens at N, E, S, W, equally far from a centre. The positions match after 90°, 180°, 270° and 360°: rotational symmetry of **order 4**. Arrow tokens must also match their orientations.

A final direction does not tell us how many turns occurred. Exact symmetry of point positions does not mean moving bodies, postures or expressions are exactly symmetrical. Our diagram is a model.

Check the mathematics, then decide the next teaching move

Worked project & rubric

Opening answer

Successive four-beat groups: **1, 2, 1, 2 taps per beat**. Their totals are **4,8,4,8**; total **24 taps within 16 beats**. Eight beats have one tap and eight have two: $8 \times 1 + 8 \times 2 = 24$.

One group = $4/16 = 1/4$. Two = $1/2$; three = $3/4$. Next group onsets are 5, 9, 13.

Ending answer

L=3, G=1. N= $3 \times 3 + 2 \times 1 = 11$ slots. Start 7. Eleven labelled positions contain **10 intervals** first-to-last.

7o 8o 9o 10-
11o 12o 13o 14-
15o 16o next1o

Play the opening from 1-16; immediately count the ending cycle from 1-16, entering on 7; finish on the following 1. From the first opening onset to the final tap is **32 intervals**, about **32 seconds at 60 BPM**. The score contains 33 labelled onsets including the start and final landing. Show the geometry diagram alongside it; a timed movement is unnecessary.

Geometry answers: 450° clockwise from N = $360^\circ + 90^\circ$, ending E. A 90° turn has the same final direction with less total turn. Reflecting (-2,1) across x=0 gives (2,1); both are two grid units from the mirror line at the same height.

Four criteria · 0-8 points

Criterion	2 · Secure	1 · Developing	0 · Next step needed
Cycle & subdivision	16 beats; four groups; correct tap total; explains what stays fixed.	Mostly correct; one count or explanation needs support.	Cycle or tap total not yet accurate.
Three-part ending	Three equal phrases, two equal gaps; labelled trace lands on next1.	Repetition clear; start, gap or final label needs a correction.	Trace does not yet explain a return to next1.
Geometry	Diagram and explanation agree about angle/direction or reflection distances.	Correct idea; a label is missing or slightly mismatched.	Transformation cannot yet be checked.
Explain & revise	Uses a calculation or diagram and records a check or correction.	Reason or check is partly supported.	Answer has no mathematical support yet.

Use diagnostically: record a next step, not a ranking. Ask an individual question where team and individual evidence differ. Do not score volume, balance, dance skill, movement, tempo accuracy or confidence. A silent paper demonstration can meet every criterion.

Useful next steps

Beats vs taps: draw two dots in one box.

One beat early: add next1 and count back.

Reflection changes height: mark horizontal distances.

Turn vs direction: compare 90° and 450°.

Possible continuation

Invite a trained Kathak practitioner to explain an authentic composition. Credit their contribution and agree on reuse. Gather classroom observations honestly; do not report this proposed activity as evidence of learning gains.

Our rhythm-and-shape project

Build the opening cycle.

Fit more taps into the same steady beat.

Name: _____ Team: _____ Date: _____

Tap, clap, point, move tokens, draw, count or explain. You can do the whole project seated and silently. Show your mathematics in your score and explanations.

1. Mark and group the cycle

Circle सम, the first beat. Divide the strip into four equal groups. Separately label the clap markers at 1, 5 and 13 and wave marker at 9.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
---	---	---	---	---	---	---	---	---	----	----	----	----	----	----	----

One group is $\frac{\quad}{16} = \frac{\quad}{\quad}$ of the cycle. Two groups are $\frac{\quad}{\quad}$. Three groups are $\frac{\quad}{\quad}$.

After half the whole cycle's duration has elapsed, beat $\frac{\quad}{\quad}$ begins.

2. Predict before trying

Taps inside each beat	1	2	3	4
Total taps in 16 beats	_____	_____	_____	_____

Keep the beat steady. Try 1 and 2 taps per beat, or show them with dots. What stays the same?

_____ What changes? _____

3. Design your opening

Use 1 or 2 taps per beat. You can begin with 1,2,1,2 in successive four-beat groups. Draw the dots **inside** each beat box above.

Four-beat group	1-4	5-8	9-12	13-16
Taps per beat	_____	_____	_____	_____
Taps in this group	_____	_____	_____	_____

Total taps = $\frac{\quad}{\quad} + \frac{\quad}{\quad} + \frac{\quad}{\quad} + \frac{\quad}{\quad} = \frac{\quad}{\quad}$.

Our cycle still has $\frac{\quad}{\quad}$ beats because _____

Peer check: [] 16 boxes [] Four equal groups [] Tap total agrees with dots.

Our correction or confirmation: _____

Our rhythm-and-shape project

Return to the next सम.

Three equal phrases. Two counted gaps. One clear landing.

Name: _____ Team: _____

Each box is one beat-position slot. o = tap at its onset; - = one silent slot. The count continues through silence. Put the last tap on the next beat 1, then work backward. This is a tihai-inspired classroom ending, not a traditional Kathak composition.

1. Build and label

Start with three taps per phrase and one silent slot between phrases. Trace the pattern onto consecutive beat positions ending at **next1**. Fill in every beat label.

Phrase 1	Gap 1	Phrase 2	Gap 2	Phrase 3
o o o	-	o o o	-	o o o
____	____	____	____	____ next1

L = ____ taps per phrase. G = ____ silent slots in each gap.

N = 3L + 2G = ____ slots. First tap on beat ____.

There are ____ beat-to-beat intervals from our first tap to our last. Why does the last tap land on next सम?

2. Test a claim

A team says: "Eleven slots, so we start on beat 6." Write all eleven beat labels. Does the final tap reach next1? Explain and correct the claim.

3. Choose and verify your team's ending

Keep L=3, G=1, or use an alternative your teacher has checked. Draw every tap, gap and beat label. Circle the final next1.

L = ____, G = ____; N = ____; start ____; final beat ____.

Peer check: [] Three equal phrases [] Two equal gaps [] Final tap at next1.

One correction or confirmation: _____

Exit challenge

L=2 taps per phrase, G=1 silent slot between phrases. How many slots? Which start reaches next1? Show the beat labels.

Our rhythm-and-shape project

Map a shape. Explain a decision.

Name: _____ Team: _____

1. Choose a geometry investigation

Work with a paper arrow or pencil dots. No body turns are needed. Choose one task; try both if time allows.

A • Rotation

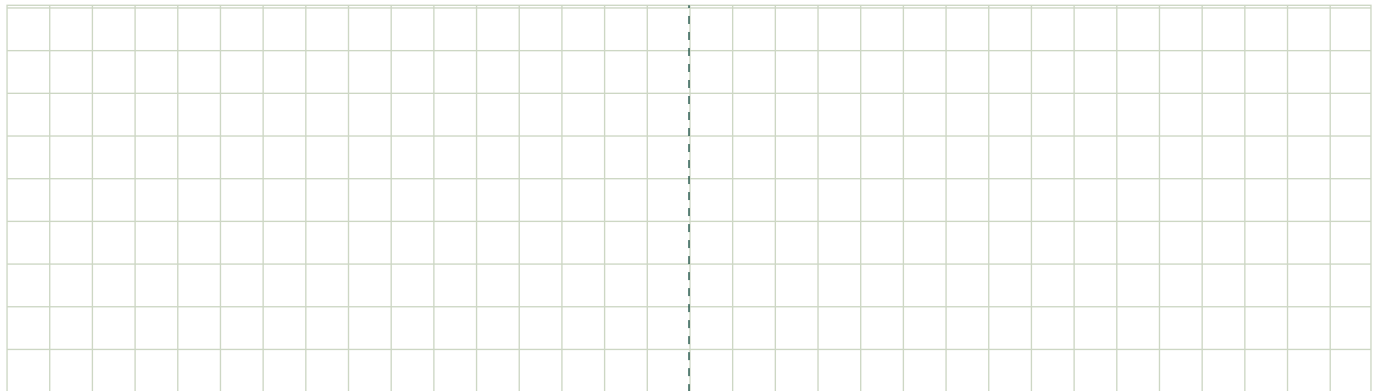
Start an arrow facing north. Turn it clockwise. Draw the start, total angle and final direction.

Find two different total turns that finish in the same direction.

B • Reflection

Use the vertical line below as a mirror. Draw two point pairs; mark equal distances to the line.

Try points two squares left / one up and one square left / two up.



Our diagram shows _____ because _____

2. Share and check

Show your opening, ending and diagram. You may point through the score silently. Ask another team to check:

☐ 16 underlying beats; four groups

☐ Correct tap total

☐ Three phrases; two internal gaps

☐ Ending trace lands on next1

☐ Diagram matches the explanation

☐ Everyone can contribute

One useful check or correction: _____

3. My individual reflection

One mathematical decision I can explain is _____ My calculation or diagram evidence is _____
 _____ One part of Kathak this project does not teach is _____
